Answer Sheet to the Written Exam

Corporate Finance and Incentives

February 2016

In order to achieve the maximal grade 12 for the course, the student must excel in all four problems.

The four problems jointly seek to test fulfillment of the course's learning outcomes: "Upon completion of the course, students will have developed an understanding of the different asset classes in financial markets as well as an approach on how to price them; including bonds, stocks, forwards, futures and options. Furthermore, the pricing methodology will also be used to illustrate how firms use these methods in order to choose their investment projects. Within the realms of Corporate Finance, we will explore the optimal capital structure of the firm, optimal dividend policy as well as how these and other factors influence how management runs the company."

Problem 1 is particularly focused on stock pricing methodology, problem 2 is particularly focused on bond pricing models, problem 3 is particularly focused on how the capital structure affects investment by the company, while problem 4 finally has a broader coverage of the learning outcomes.

Some numerical calculations may differ slightly depending on the software used for computation, so a little slack is allowed when grading the answers.

Problem 1 (APT 25%)

1) Let the portfolio have weights $x^T = (x_1, \ldots, x_4)$ which sum to one. The expected return on the portfolio is $x^T r$ where r is the vector of stock returns. From the factor model, let us define the matrix

$$\hat{B} = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & -1 & 2 & 1 \\ 3 & 2 & 0 & 1 \\ 4 & -3 & 1 & 1 \end{bmatrix}.$$

The goal is then to find the vector x that solves $x^T \hat{B} = (0, 0, 0, 1)$. Solving this matrix equation gives the solution $x^T = (-14.5, 8, 9, -1.5)$. The return on this portfolio is $r_f = x^T r = .015$.

2) Here we need to solve $x_i^T \hat{B} = \left(e_i^T, 1\right)$ where $e_i \in \mathbb{R}^3$ is the unit vector for coordinate i, for $i = 1, 2, 3$. The solutions are: Factor 1, $x_1^T = (-11, 6, 7, -1)$; factor 2, $x_2^T = (-12.5, 7, 8, -1.5)$; factor 3, $x_3^T = (-8, 5, 5, -1)$.

3) Expected returns to the four stocks are $\bar{r}^T = (.07, .02, .09, -.04)$. The factor risk premium is $\lambda_i = x_i^T \bar{r} - r_f$. Compute $\lambda_1 = .005$, $\lambda_2 = .03$, $\lambda_3 = .015$.

4) Collecting pieces from above, we find: stock 1, $2\lambda_1 + \lambda_2 + \lambda_3 = .055 = .07 - .015$; stock 2, $\lambda_1 - \lambda_2 + 2\lambda_3 = .005 = .02 - .015$; stock 3, $3\lambda_1 + 2\lambda_2 + 0\lambda_3 = .075 = .09 - .015$; stock 4, $4\lambda_1 - 3\lambda_2 + \lambda_3 = -.055 = .04 - .015$.

Problem 2 (Fixed Income 25%)

1) We need vector $d^T = (d_1, d_2, d_3)$ to solve $\pi = Cd$. Solving this matrix equation gives $d^T = (.902, .930, .884)$.

2) The t-year yield $y(0,t)$ solves $d_t (1 + y(0,t))^t = 1$. We get approximately $y(0,1) = 10.85\%$, $y(0,2) = 3.70\%$, $y(0,3) = 4.20\%$.

3) The Macaulay duration for bond n is $D_n = \left(\sum_{t=1}^{3} c_{nt} d_t t\right) / \pi_n$. This gives approximately $D_1 = 2.83$, $D_2 = 2$, and $D_3 = 1.45$.

Problem 3 (Investment Opportunity 25%)

1) All amounts are given in million Kroner. In the high state, cash-flow to the firm is 130, and equity holders get 30. In the low state, the firm gets 110 and equity holders get 10. Compute the expected present values of these cash flows. The value of the firm is approximately

$$V = \frac{40\%130 + 60\%110}{1.03} = 114.56,$$

and the value of its equity is approximately

$$E = \frac{40\%30 + 60\%10}{1.03} = 17.48.$$

2) The firm value is constant because the cash-flow change at scale Z has present value $(40\%60Z + 60\%40Z)/1.03 = 0$ The critical point arises where Z gets so large that bond holders no longer can get all of 100 million Kroner in the low state. This point solves $110 - 40Z^* = 100$, that is $Z^* = 25\%$. For $Z < Z^*$ bond holders are indifferent because they still get the same, 100 in each state. For $Z > Z^*$, bond holders get less in the low state (but not more in the high state), so they lose. Since the firm value stays constant, equity holders gain.

3) The scale Z can grow so large that bond holders no longer lose from augmentations to the project. This happens at the point when they simply get no payoff in the low state. Thus, Z^{**} solves $110 - 40Z^{**} = 0$, or $Z^{**} = 275\%$.

Problem 4 (Various Themes 25%)

1) See the textbook's Section 13.6.

2) The textbook's chapter 18 assumes from the outset that (i) the project has the same risk as the firm, (ii) the debt-equity ratio is kept constant, and (iii) corporate taxes are the only capital market imperfection.

3) See the textbook's discussion of "Limits to the Tax Benefit of Debt" in Section 15.5.

4) See the textbook's Sections 20.1–2 for the definition, and chapters 20–21 for the relevant factors in the Black-Scholes formula.